8.—*The Life and Works of A. K. Erlang.* By E. Brockmeyer, H. L. Halstrøm and Arne Jensen. Copenhagen: The Copenhagen Telephone Company, 1948. 277 pp. 9¾". (*Trans. Danish Acad. Techn. Sci.* (1948), No. 2.)

This co-operative work has been published by the Directors of the Copenhagen Telephone Company in memory of the late A. K. Erlang, and to celebrate the adoption of "the erlang" as the international unit of telephone traffic.

Erlang (who was born in 1878) joined the Company in 1908 as scientific collaborator and head of its laboratory. The appointment could not have been more timely; in the preceding year Fr. Johannsen (the managing director) had published two essays on the application of the calculus of probabilities to the mathematical problems associated with waiting-time and busy-signal systems, and he immediately suggested to Erlang that he should take over this work and develop it in a systematic way. A year later appeared Erlang's first paper dealing with these topics, and from then until his death in 1929 he remained the acknowledged world expert on the subject. Some indication of the pioneer quality of Erlang's work can be given by noting that his most important paper (1917) was eventually published in four different journals in as many languages. Two other workers in the field, A. E. Vaulot and T. C. Fry, are said to have taught themselves Danish in order to read Erlang's papers in the original.

The first essay in this book (by Brockmeyer and Halstrøm) contains an account of Erlang's life. Several portraits and a note of his "heavy red full beard" suggest a striking personality. There then follows a long and valuable paper by Arne Jensen giving a unified account of Erlang's investigations from the standpoint of the modern theory of stochastic processes. This forms a very useful supplement to the collection of Erlang's own papers (translated into English), which are most interesting, but are not too easy to read by reason of the unfamiliarity of the notation and the occasional suppression of the mathematical working. Here will be found his original solutions to the now well-known waiting-time and probability-of-loss problems. A graph showing the distribution in duration of some 2,500 calls dealt with by Copenhagen main exchange enables

one to judge the adequacy of the negative-exponential distribution often employed in the theoretical calculations. Erlang most frequently used this or the assumption of a constant holding-time, but he also invented the idea of a multi-stage conversation leading theoretically to a distribution for the holding-time of χ^2-form with an even number of degrees of freedom (which it has more recently been of interest to consider in connection with bacterial generation-times).

Erlang's own papers contain a number of tables of the mathematical functions required (including one for a Poisson distribution with negative mean), and the present volume is completed by Brockmeyer's table of Erlang's loss formula.

Telephone traffic is now said to have an intensity of m erlangs if m calls are expected during an interval equal to the mean holding-time. The quantity thus measured is of course dimensionless, and the erlang is to be compared with the octave, the stellar magnitude and the decibel in describing the mode of calculation rather than the unit of measurement in the usual sense of physics.

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9.—*Moe's Principle.* By Arne Jensen. Copenhagen: The Copenhagen Telephone Company, 1950. 160 pp. Charts. $9\frac{3}{4}''$.

The late K. Moe, Engineer-in-Chief to the Copenhagen Telephone Company, who in the course of his working at the operational economy of telephone exchanges occupied himself much with the Erlangian loss and waiting-time formulae, put forward in 1923 a rational method of balancing against the expense involved the advantage gained by introducing an additional connecting device in a telephone system. This caused great controversy, even among his colleagues, but "Moe's Principle" steadily gained ground, and just before Moe's death (in 1949) Arne Jensen was invited by the Company to produce the present work, which is a treatise on the rational application of the theory of probability to the management of telephone plant. The theory is based on "Moe's Principle", and the volume includes an extensive set of tables which is intended to facilitate the application of Moe's methods to practical problems.

Very briefly, the principle asserts that the decrease in the number of lost calls (or of waiting-time units) per unit of time obtained by adding one circuit to the group must be the same whether the number of circuits in the group is large or small.

Jensen's memoir commences with an account of the relevant parts of econometric theory, and then proceeds to derive a set of rules governing the optimum lay-out of a telephone system. The formulation is very general, and allows the optimum solution to be chosen in accordance with a wide variety of economic aims. (Thus it is possible to take account of the financial loss suffered by the subscriber during his waiting-time.) The mathematical details are rather complicated; in part this is a consequence of the discrete character of the independent variables (which implies that "differential" have to be replaced by "difference" methods throughout) but it is also partly caused by the fact that the optimum solution will frequently correspond to a boundary-point of the region of variation so that a stationary solution, even if it exists, may easily be the wrong one.

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